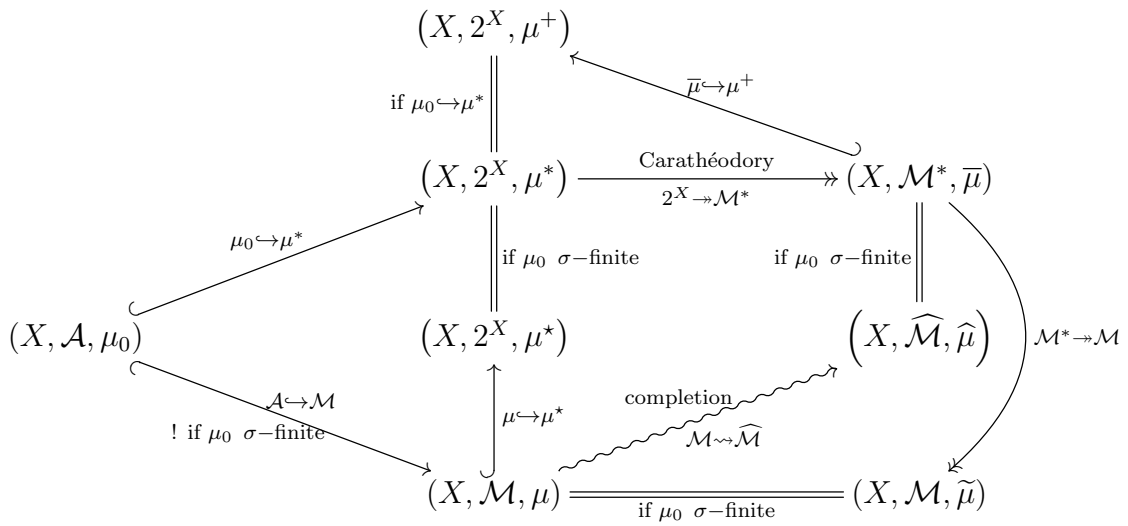
The Construction of Abstract Measures

Designed by *Fraljimetry* on August 22, 2024

Acknowledgement This diagram was derived from Chapter 1 of *Real Analysis: Modern Techniques and Their Applications (Second Edition)* by Gerald B. Folland.

Construction



Comments on Ambiguities

- X is any given set and 2^X is the power set of X.
- $\mathcal{A} \subset 2^X$ is an algebra and $\mu_0 : \mathcal{A} \to [0, \infty]$ is a premeasure on $\mathcal{A}$.
- $\mathcal{M} \subset 2^X$ is the σ-algebra generated by $\mathcal{A}$. The measure μ is unique if μ_0 is σ-finite.
- $\mu_0 \hookrightarrow \mu^*$, $\mu \hookrightarrow \mu^\star$, and $\overline{\mu} \hookrightarrow \mu^+$ are all extensions to outer measures, while two head arrows $\twoheadrightarrow$ are restrictions.

Significance

- That μ_0 being σ-finite is a crucial condition to prove many relations above, which will largely simplify the diagram.
- The premeasure space $(X, \mathcal{A}, \mu_0)$ is extremely rough and general, but the ultimate complete measure space $(X, \mathcal{M}^*, \overline{\mu})$ behaves nicely and will be used throughout all branches of analysis.